Lattice-Boltzmann Models of Ion Thruster Cathode 3D MHD Flows

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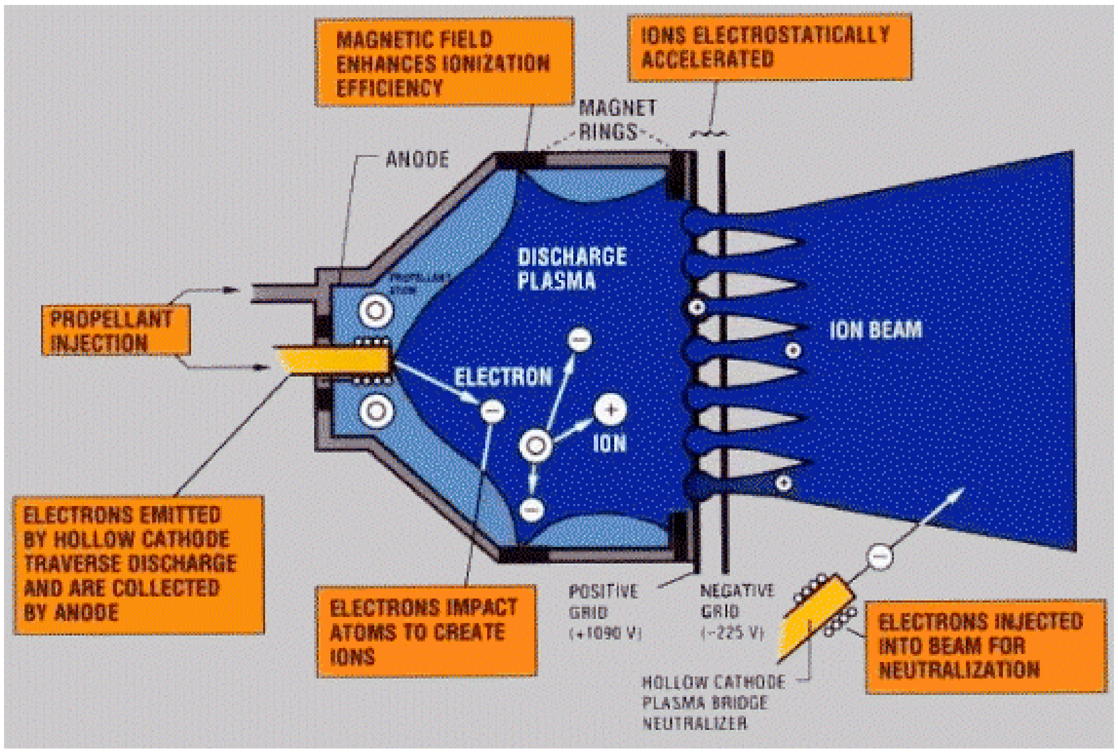
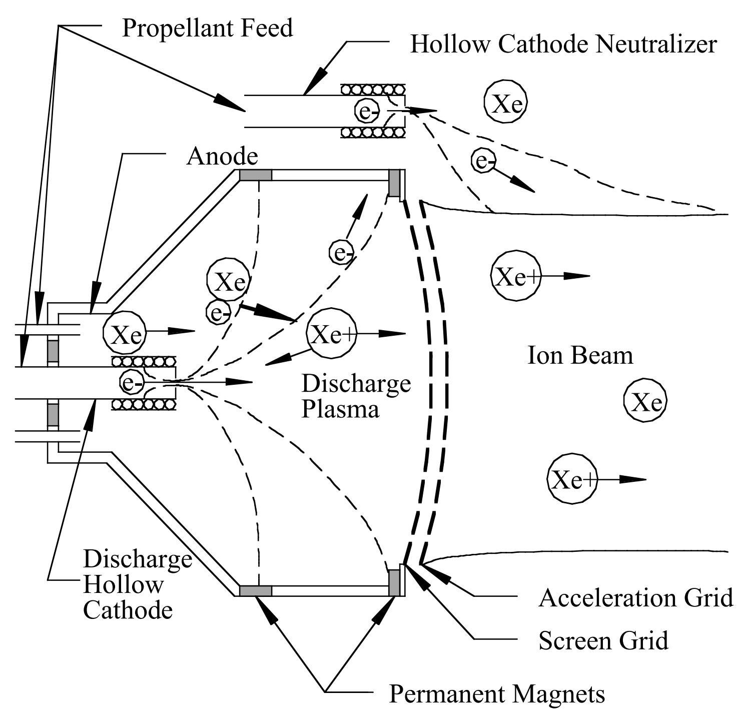
Ion thrusters (see Gallimore, 2004)



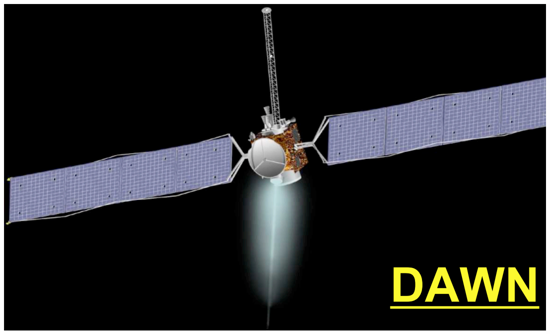


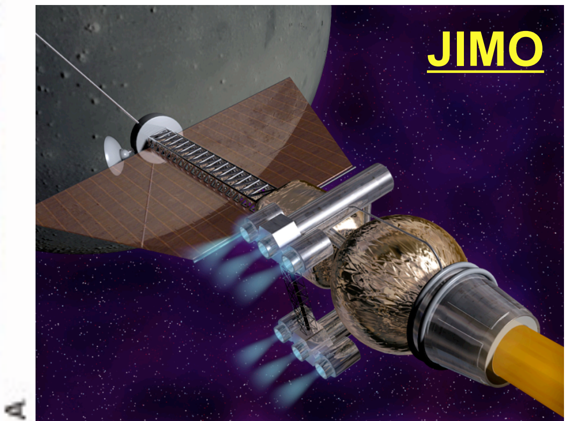
Ion thrusters are the most efficient EP devices at converting input power to thrust and are used both as primary propulsion and for station-keeping on commercial and scientific spacecraft.
Key issues include grid erosion and thrust density limitations from space-charge effects.

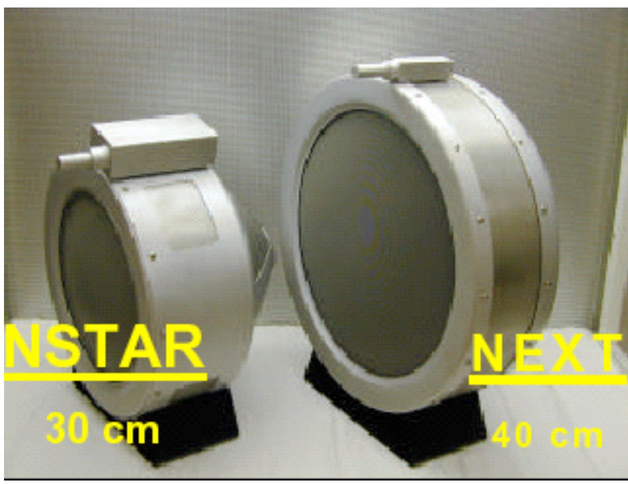
Modern Ion Thrusters

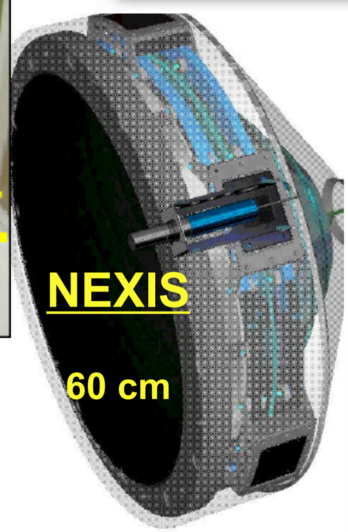
Solar Electric Propulsion — NASA's Evolutionary Xenon Thruster (NEXT) [5-10 yr. deployment time]

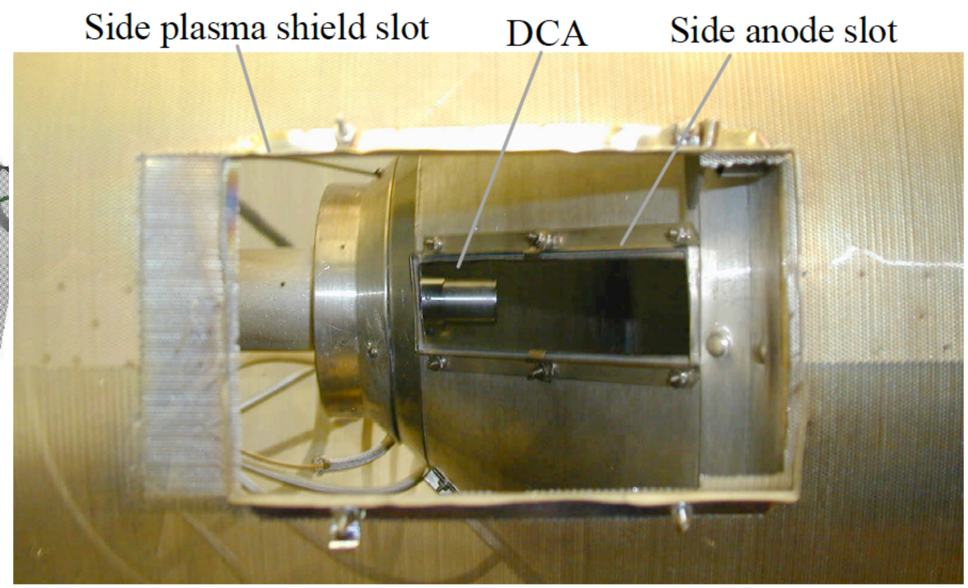
Nuclear Electric Propulsion — NASA's Nuclear Space Initiative [10-15 yr. deployment time]











Typical Ion Engine Parameters

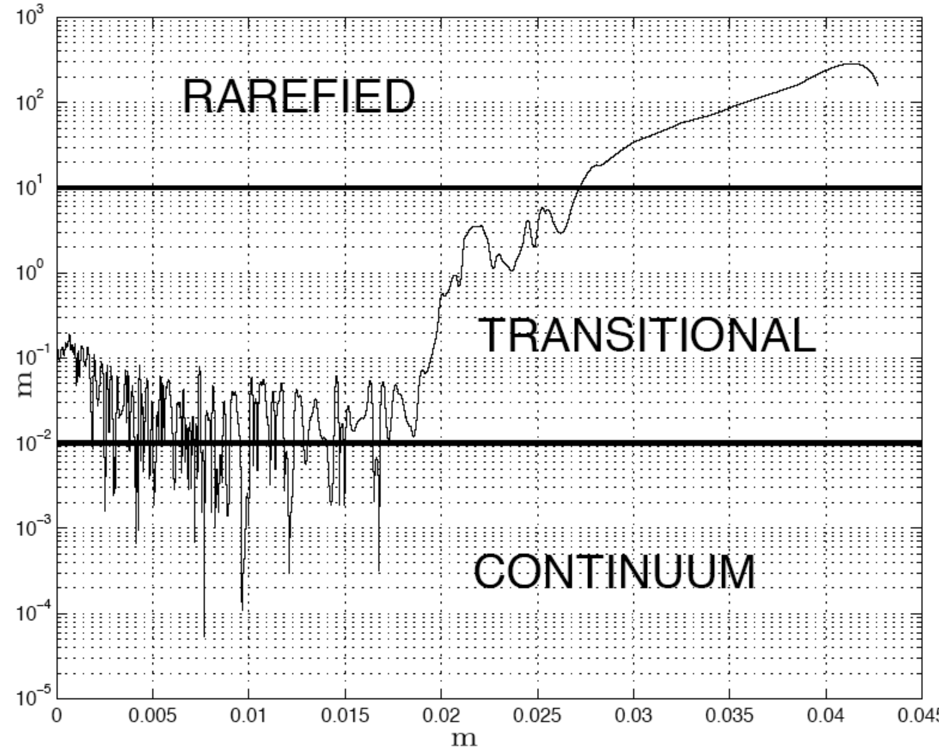
- Near DCA & grid, typical ion thruster & plasma parameters are:

  n_{Xe+} ~ 10^{13}-10^{10} cm^{-3} , n_e ~ 10^{13}-10^{10} cm^{-3} >> n_{Xe} >> n_{Xe++} …
- V_+ ~ 1075 V at screen grid
- V_- ~ -150 V at accelerator grid
- Grid separation ~ 1 mm
- Screen grid opening diameter ~ 2 mm
- Accelerator grid opening diameter ~ 1 mm
- DCA V_{dc} = 25V difference from anode
- B = 100G

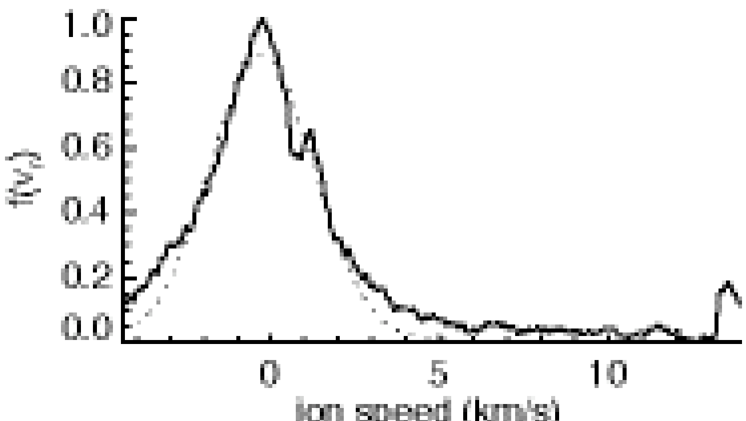
Assumptions of Applicability of LB

- Note local Kn:

Crawford (2002)



- $Kn \sim O$(0.1) around optics & DCA
- Maxwellian radial $f(\mathbf{v})$



$$Kn = \frac{\lambda}{L} = \frac{RT}{\sqrt{2}\pi d^2 N_A p L}$$

$$L = \frac{\rho}{d\rho / dx}$$

Ion veloc. distrib. Laser-induced Fluorescence Velocimetry of Xe II in the 30-cm NSTAR-type Ion Engine Plume, Smith and Gallimore (AIAA-2004-3963)

MHD Equations

- MHD eqs. comprise conservation of
  - mass,
  - momentum,
  - magnetic flux
  - energy
  - where

$$\frac{\partial}{\partial t}\begin{bmatrix}\rho \\ \rho\mathbf{v} \\ \mathbf{B} \\ E\end{bmatrix} + \nabla\bullet\begin{bmatrix}\rho\mathbf{v} \\ \rho\mathbf{v}\mathbf{v} + p\mathbf{I} - \frac{1}{\mu}\left(\mathbf{B}\mathbf{B} - \frac{1}{2}B^2\mathbf{I}\right) \\ \mathbf{v}\mathbf{B} - \mathbf{B}\mathbf{v} \\ (E+p)\mathbf{v} - \frac{1}{\mu}\left(\mathbf{B}\mathbf{B} - \frac{1}{2}B^2\mathbf{I}\right)\bullet\mathbf{v}\end{bmatrix} = \nabla\bullet\begin{bmatrix}0 \\ \tau_{vis} \\ \mathbf{E}_{resist} \\ \mathbf{q}\end{bmatrix}$$

$$-\nabla\bullet\mathbf{E}_{resist} = \nabla\times\left(\frac{1}{\sigma}\mathbf{j} + \frac{1}{en_e}\mathbf{j}\times\mathbf{B}\right) = \left[\eta_0\vec{\vec{j}}\right] - \left[\vec{\vec{\eta}}\bullet\vec{\vec{j}}\right] + \left[\vec{\vec{\eta}}\bullet\vec{\vec{j}}\right]^T$$

Sankaran (2001)

LBM Basics

- The molecular velocity distribution function $f = f(x_i, c_i, t) = f(\mathbf{x}, \mathbf{c}, t)$ has a rate of change, w.r.t. position & time,

$$\frac{\partial}{\partial t}[nf(c_i)] + c_j \frac{\partial}{\partial x_j}[nf(c_i)] = \left\{\frac{\partial}{\partial t}[nf(c_i)]\right\}_{collision}$$

$$\left\{\frac{\partial}{\partial t}[nf(c_i)]\right\}_{collision} = \int_{-\infty}^{\infty}\int_{0}^{2\pi}\int_{0}^{\pi/2} n^2 \mathrm{d}^2 [f(c'_i)f(z'_i) - f(c_i)f(z_i)] g \sin\psi \cos\psi d\psi d\varepsilon dV_x$$

- Macroscopic variables of interest

$$\rho = \int_{-\infty}^{\infty} mf dV_c \qquad \rho\mathbf{u} = \int_{-\infty}^{\infty} m\mathbf{c} f dV_c \qquad e_{tr} = \int_{-\infty}^{\infty} \frac{1}{2} m(\mathbf{c} - \overline{\mathbf{c}})^2 f dV_c$$

The "1st Order" Boltzmann Equation

- So Boltzmann equation becomes, to 1st order:

$$\frac{\partial f}{\partial t} + \mathbf{c} \cdot \nabla f = -\frac{1}{\lambda}\left(f - f^{(eq)}\right) = \frac{df}{dt}$$

- On the characteristic line $\mathbf{c} = d\mathbf{x}/dt$, where $\lambda = 1 / \nu_r$
- Integrating over a small time step δ_t, assuming $f^{(eq)}$ is smooth enough locally, and expanding in a Taylor series while neglecting terms of $O(\delta_t^2)$ and also defining $\tau = \lambda/\delta_t$:

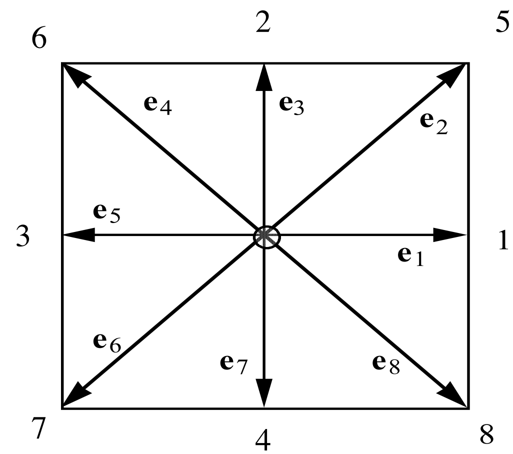
$$f(\mathbf{x} + \mathbf{c}\delta_t, \mathbf{c}, t + \delta_t) - f(\mathbf{x}, \mathbf{c}, t) = -\frac{1}{\tau}\left[f(\mathbf{x}, \mathbf{c}, t) - f^{(eq)}(\mathbf{x}, \mathbf{c}, t)\right]$$

2D Square Lattice-Boltzmann Model 9-bit

- Cartesian coord. moment integral of $\psi_{mn}(\mathbf{c}) = c_x^m c_y^n$

$$I_m = \frac{m}{\pi}\sum_{i,j=1}^{3}\omega_i\omega_j\psi\left(c_{i,j}\right)\left\{1+\frac{c_{i,j}\bullet\mathbf{u}}{kT}+\frac{\left(c_{i,j}\bullet\mathbf{u}\right)^2}{2\left(kT\right)^2}-\frac{\mathbf{u}^2}{2kT}\right\}$$

- Select possible molecular velocities on 2D square lattice to go in as many directions of such square
- The chosen velocities have a certain symmetry to account for molecules moving in any and all directions independent of directions (isotropy)
- For square this means sides & corners
- This means 9 possible velocities
- To go w/at least 9 terms in $\psi_{mn}(\mathbf{c})$



Lattice Boltzmann Equation

- Equilibrium distribution function for f_α

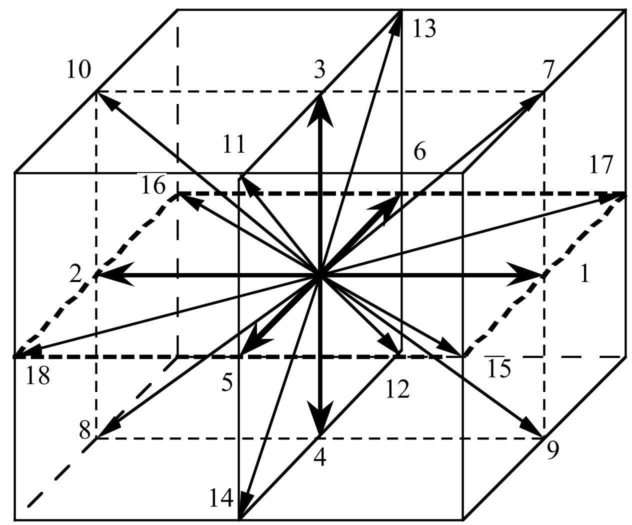
$$f_\alpha^{(eq)} = \rho w_\alpha [1 + \frac{3}{c^2}\boldsymbol{e}_\alpha \cdot \boldsymbol{u} + \frac{9}{2c^4}(\boldsymbol{e}_\alpha \cdot \boldsymbol{u})^2 - \frac{3}{2c^2}\boldsymbol{u}\cdot\boldsymbol{u}]$$

Δ 9-velocity model(2D):

$$w_\alpha = \begin{cases} 4/9, & \alpha = 0 \\ 1/9, & \alpha = 1,3,5,7 \\ 1/36, & \alpha = 2,4,6,8 \end{cases}$$

Δ 19-velocity model (3D):

$$w_\alpha = \begin{cases} 1/3, & \alpha = 0 \\ 1/18, & \alpha = 1,2,\ldots 6 \\ 1/36, & \alpha = 7,8,\ldots 18. \end{cases}$$



Q19D3 lattice

Qian YH, d'Humieres D, Lallemand P. Lattice BGK Models for Navier Stokes Equation. Europhys Lett 1992;17:479-484.

LBM MHD EP Model

- The model assumes coupling of the velocity distribution function w/the Lorentz force $\mathbf{j}\times\mathbf{B}$ in acceleration $\mathbf{a}$

$$\frac{\partial f}{\partial t} + \mathbf{c}\bullet\nabla_{\mathbf{r}} f + \mathbf{a}\bullet\nabla_{\mathbf{c}} f = Q(f,f)$$

- Then

$$f(\mathbf{x}+\mathbf{c}\delta_t,\mathbf{c},t+\delta_t) - f(\mathbf{x},\mathbf{c},t) = -\frac{1}{\tau}\left[f(\mathbf{x},\mathbf{c},t) - f^{(eq)}(\mathbf{x},\mathbf{c},t)\right] - \frac{\delta_t}{2}\mathbf{a}\bullet\nabla_{\mathbf{c}} f$$

LBM MHD EP Model

- Vector distrib. $\mathbf{g}(\mathbf{x}, \mathbf{w}, t)$ gives $\mathbf{B} = \int \mathbf{g}\, d\mathbf{w}$
- Macroscopic MHD eqs. also recovered
- Evolution of $\mathbf{g}(\mathbf{x}, \mathbf{w}, t)$ obeys a kinetic eq

$$\frac{\partial \mathbf{g}}{\partial t} + \mathbf{w} \cdot \nabla \mathbf{g} = -\frac{1}{\lambda_m}\left(\mathbf{g} - \mathbf{g}^{(eq)}\right) = \frac{d\mathbf{g}}{dt}$$

$$\mathbf{g}(\mathbf{x} + \mathbf{w}\delta_t, \mathbf{w}, t + \delta_t) - \mathbf{g}(\mathbf{x}, \mathbf{w}, t) = -\frac{1}{\tau_m}\left[\mathbf{g}(\mathbf{x}, \mathbf{w}, t) - \mathbf{g}^{(eq)}(\mathbf{x}, \mathbf{w}, t)\right]$$

$$\rho = \sum_{\alpha=1}^{N} f_\alpha \qquad \rho\mathbf{u} = \sum_{\alpha=1}^{N} \mathbf{e}_\alpha f_\alpha + (\mathbf{j} \times \mathbf{B})\frac{\delta_t}{2} \qquad \mathbf{B} = \sum_{\beta=1}^{M} \mathbf{g}_\beta$$

Dellar, P., “Lattice Kinetic Schemes for MHD”, Journal of Computational Physics **179,** 95–126 (2002).
G. Breyiannis and D. Valougeorgis, “Lattice kinetic simulations in three-dimensional magnetohydrodynamics”, PHYS. REV. E **69**, 065702(R) (2004)

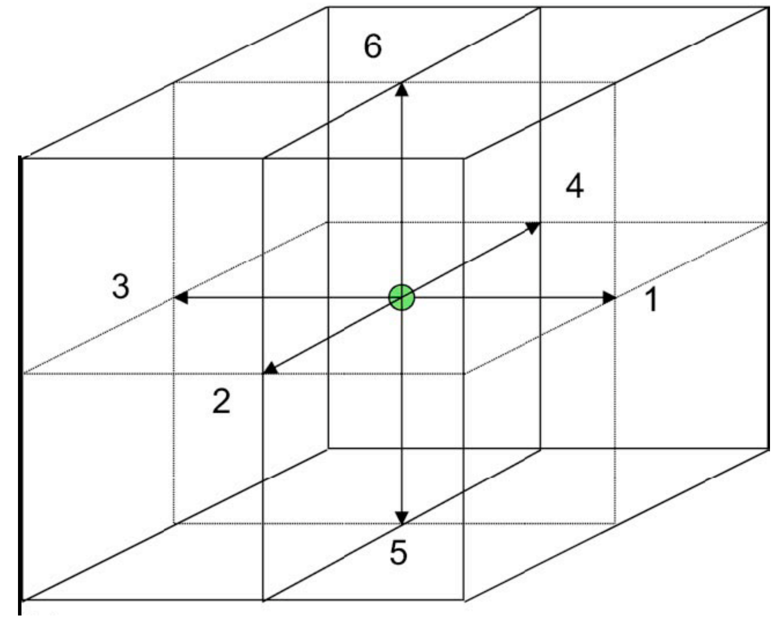
LBM MHD Parameters

- The magnetic resistivity is $\eta = \tau_m \theta_m \delta_t$
- where θ_m=1/4
- Constraints are

$\Sigma_\beta W_\beta = 1$, $\Sigma_\beta W_\beta \zeta_{j\beta} \zeta_{\beta j} = 0$

- For lattice to retain symmetry up to 3rd order:

$\Sigma_\beta W_\beta \zeta_{\beta\alpha}$=0, $\Sigma_\beta W_\beta \zeta_{\beta\alpha} \zeta_{\beta j} \zeta_{\beta\gamma}$=0

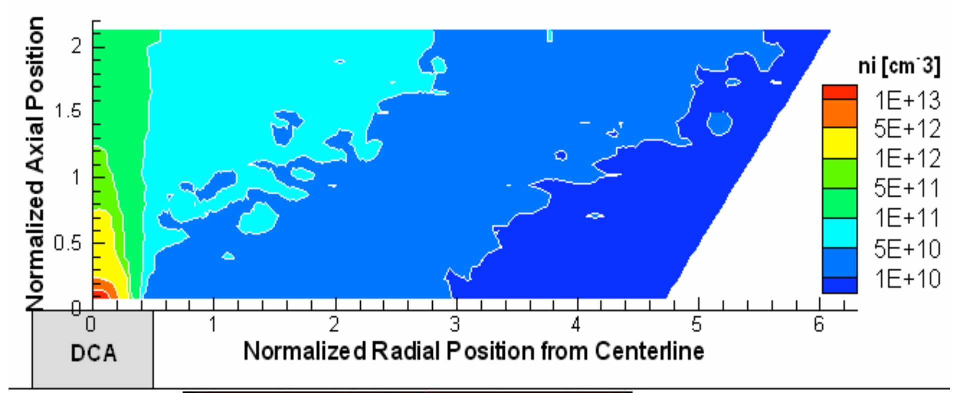


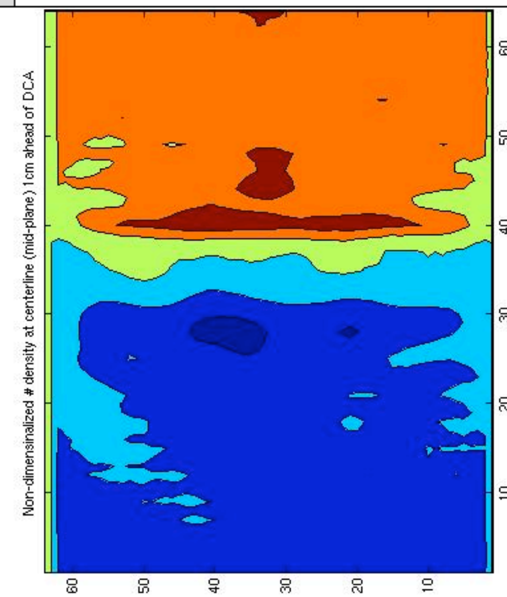
$$g_{\beta j}^{(eq)} = W_\beta \left[B_j + \theta_m^{-1} \zeta_{\beta i} \left(u_i B_j - B_i u_j \right) \right]$$

$$W_\beta = \begin{cases} 1/4, & \beta = 0 \\ 1/2, & \beta = 1,2,\ldots 6 \end{cases}$$

Electron/Ion number density mappings

- NASA Solar Electric Propulsion Technology Applications Readiness (NSTAR) 30-cm ion thruster
- Thruster operating conditions: $V_{dc} = 25.10$ V and $J_{dc} = 8.24$ A
- LBMHD #density nozzle expansion characteristics partly captured
- But not range of #s

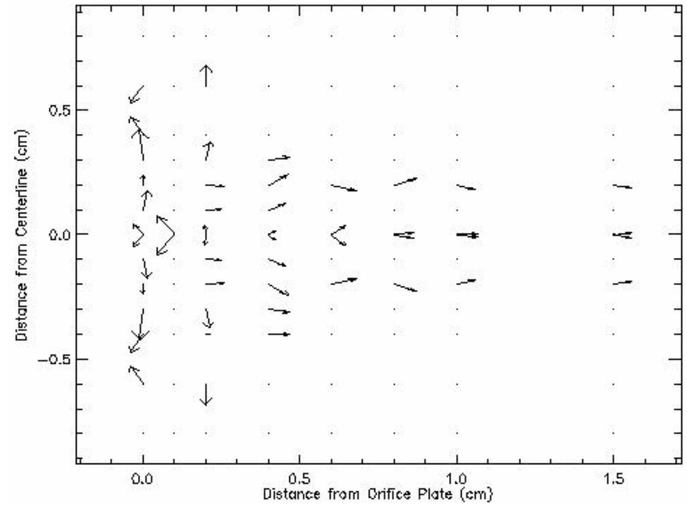


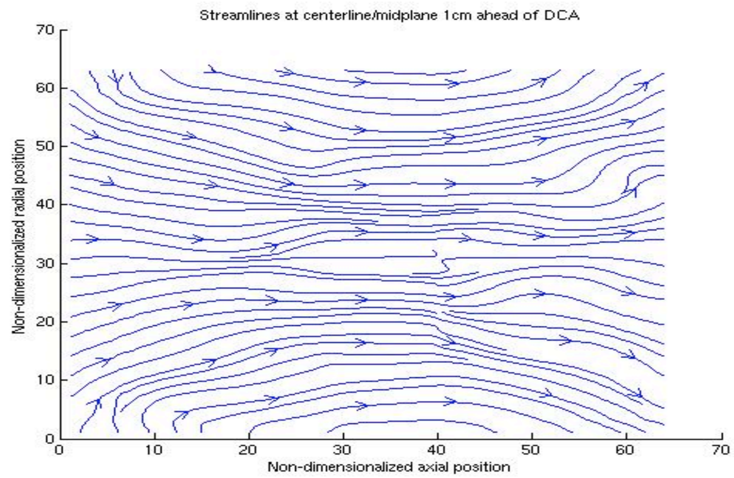


Daniel A. Herman† and Alec D. Gallimore, "Discharge Chamber Plasma Structure of a 30-cm NSTAR-type Ion Engine"
Plasmadynamics and Electric Propulsion Laboratory, U. Michigan, Ann Arbor, MI 48109 USA, AIAA-2004-3794

Velocity map

- Xe II velocity map for 12 A, 27 V operation.
- Note regions of back-flowing ions along the cathode face (x=0) & region of small velocities along centerline between 0.3 & 0.6 cm downstream.
- Some individual species effects on plasma stream at centerline & edges
- LBMHD captures major plasma flow, not back-flow details

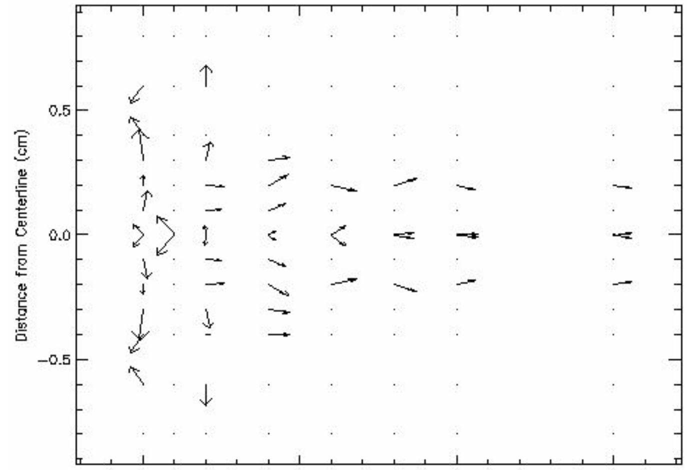


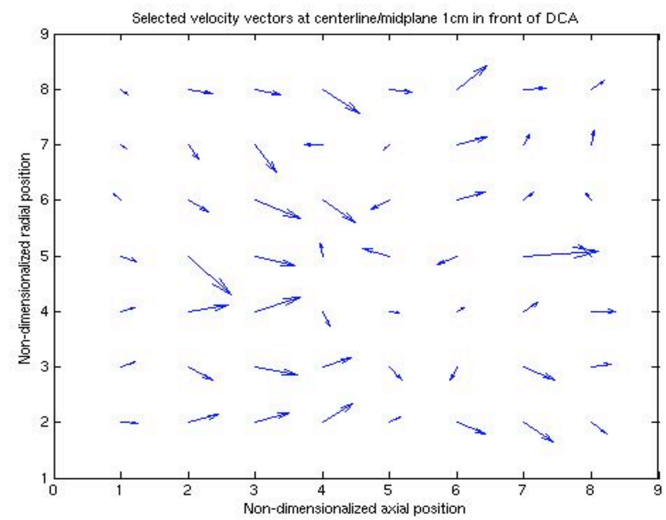


G. J. Williams, Jr., T. B. Smith, K. H. Glick, Y. Hidaka, and A. D. Gallimore, "FMT-2 Discharge Cathode Erosion Rate Measurements via Laser-Induced Fluorescence", AIAA-00-3663

Velocity map

- Xe II velocity map for 12 A, 27 V operation.
- Note regions of back-flowing ions along the cathode face (x=0) & region of small velocities along centerline between 0.3 & 0.6 cm downstream.
- LBMHD w/a little more plasma turbulence captures more of the flow trends and some back-flow





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Conclusions & Future Work

- LBMHD does OK modeling EP bulk fluid flow
- Other species back-flow too buried in MHD plasma model
- Electrostatic single species model does better as in LBM-Poisson model of optics *
- Next is to try
  - other species, perhaps quasi-3D w/Maxwell's eqs.
  - variations in collision operator, e.g.,pseudo-random collision frequency as in DSMC
  - Other variations of BE form

* Richard, J. C. and Shah, P. Application of LBM to Xe^+ Flow about Ion Thruster Optics", Int'l. Conf. for Mesoscopic Methods in Engineering and Science (ICMMES), Technical University of Braunschweig, Germany, July 26 - 29, 2004 www.icmmes.org