# Magnetic Field Effects on Axis-Switching and Instabilities in Rectangular Plasma Jets

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# Outline

- Motivation
- Rectangular jets
- Magnetohydrodynamics
- Lattice Boltzmann Method for Magnetohydrodynamics
- MHD Rectangular Jets
- Conclusion

# Motivation

- High-Speed Flow Control, Space Propulsion, Materials Processing, Jet casting
- Even nano-tube processing (Poisson-Boltzmann eqs., Sigmund *et al*. [2000])
- Previous works used B fields for flow control of channel flows, scramjet inlets, hypersonic boundary layers
- Most previous works: 2D or axi-symmetric jet; uniform, transverse B fields

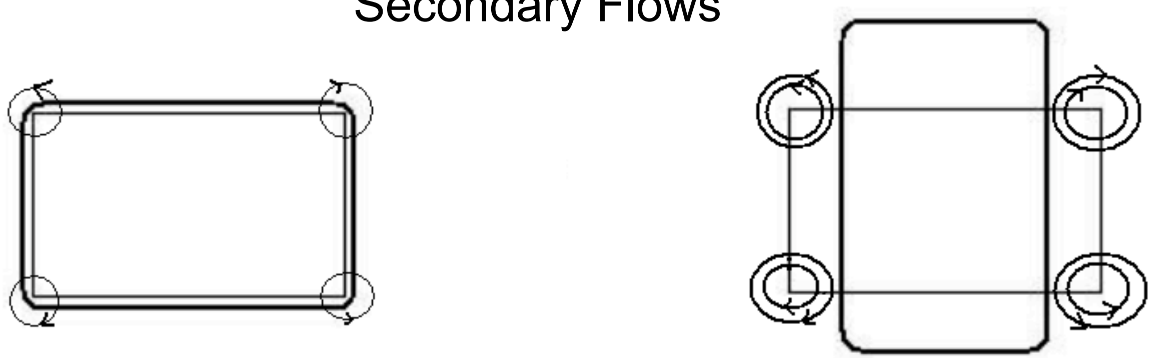
# Current Work

- 3D RJ, axi-symmetric magnet loop
- More flow control options than just jet deceleration
- Loop used in space propulsion (MPD thrusters, VASIMR, magnetic nozzle)

# Rectangular Jet Flow Characteristics

- Rectangular Jet’s (RJ) are inherently unstable.
- Secondary flows cause minor axis shear layer growth to exceed that of the major axis.
- This instability (Kelvin Helmholtz) causes unsteadiness and axis-switching.
- Makes RJ’s more easily manipulated for flow control.

Secondary Flows



No Axis-Switching at Low Reynolds numbers

Axis-Switching at Higher Reynolds number

Picture is view of the streamwise plane (Y-Z) of the jet flow

Gutmark EJ and Grinstein FF [1999]

# Magnetohydrodynamics

Navier-Stokes & Maxwell's eqs. lead to MHD eqs. subject to key assumptions

Assume quasi-charge neutrality, two-fluid eqs. (ions & e-) can be simplified to single-fluid.

Cyclotron period << hydrodynamic processes' relaxation times

Radii, Debye length << hydrodynamic length scales

$$\frac{\omega_{pe}^{-1}}{\tau_H} \ll 1 \qquad \frac{\lambda_D}{\lambda_H} \ll 1 \qquad \frac{R_e}{\lambda_H} \ll \frac{R_i}{\lambda_H} \ll 1$$

# Magnetohydrodynamic Eqns.

The resulting magnetohydrodynamic equations are:

$$\rho \frac{\partial \mathbf{v}}{\partial t} + \rho \mathbf{v} \cdot \nabla \mathbf{v} = -\nabla p - \nabla \left( \frac{B^2}{2\mu_0} \right) + \frac{1}{\mu_0} (\mathbf{B} \cdot \nabla) \mathbf{B} + \mu \nabla^2 \mathbf{v}$$ (LM)

$$\frac{\partial \mathbf{B}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{B} = \mathbf{B} \cdot \nabla \mathbf{v} + \frac{\eta}{\mu_0} \nabla^2 \mathbf{B}$$ (Magnetic induction)

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$ (Mass Conservation)

$$\nabla \cdot \mathbf{B} = 0$$ (Solenoidal **B**, no magnetic monopoles)

# MHD Characteristic Parameters

- **Magnetic Reynolds Number**: $R_m = \frac{\mu_0 v L}{\eta} = \frac{vL}{\sigma} \Rightarrow \frac{convection}{diffusion} = \frac{\mu_0\left[(\mathbf{v}\cdot\nabla)\mathbf{B} - (\mathbf{B}\cdot\nabla)\mathbf{v}\right]}{\eta\nabla^2\mathbf{B}}$

$R_m \ll 1$ magnetic field growth slowed

$R_m \gg 1$ magnetic field growth coupled to velocity

- **Ratio of kinetic to magnetic energy**: $\beta = \frac{2\mu_0 \rho v^2}{B_0^2} \Rightarrow \frac{\text{Kinetic Energy}}{\text{Magnetic Energy}}$

$\beta \gg 1$ kinetic energy dominates motion

$\beta \ll 1$ magnetic energy "

- **Interaction Parameter**: $N = \frac{\sigma B^2 l}{\rho u} \Rightarrow \frac{\text{Lorentz Force}}{\text{inertial Force}}$

$N \ll 1$ inertial forces dominate formation of coherent structures

$N \gg 1$ Lorentz forces "

- **Hartmann Number**: $H = \frac{B_0 L}{\sqrt{\rho_0 \eta v}} \Rightarrow \frac{\text{Lorentz Force}}{\text{Viscous Force}}$

$H \ll 1$ viscous forces dominate boundary layer growth

$H \gg 1$ Lorentz forces "

Note: $H^2 = N\,\mathrm{Re} = Ch$ = Chandrasekhar #

# MHD LBM Formulation

$$\frac{\partial f_\alpha}{\partial t} + \mathbf{v}\cdot\frac{\partial f_\alpha}{\partial \mathbf{r}} + \frac{q_\alpha}{m_\alpha}(\mathbf{E}+\mathbf{v}\times\mathbf{B})\cdot\frac{\partial f_\alpha}{\partial \mathbf{v}} = \left(\frac{\partial f_\alpha}{\partial t}\right)_{coll}$$

Eq for density distribution function using BGK collision operator, neglecting Lorentz force

$$\frac{\partial f_\alpha}{\partial t} + \mathbf{c}\cdot\frac{\partial f_\alpha}{\partial \mathbf{r}} = -\frac{1}{\tau}\left(f_\alpha - f_\alpha^{(eq)}\right)$$

$$f_\alpha(x+e\delta_t, t+\delta_t) - f_\alpha(x,t) = -\frac{1}{\tau}\left[f_\alpha(x,t) - f_\alpha^{(eq)}(x,t)\right]$$

Dellar [2002] equilibrium distribution function used.

$$w_\alpha\rho\left(1+\frac{3}{c^2}\boldsymbol{e}_\alpha\cdot\mathbf{v}+\frac{9}{2c^4}(\boldsymbol{e}_\alpha\cdot\mathbf{v})^2 - \frac{3}{2c^2}\mathbf{v}\cdot\mathbf{v}\right)+\frac{9}{2c^2\mu_0}w_\alpha\left(\frac{1}{3}|B_\alpha|^2|e_\alpha|^2-(\mathbf{e}_\alpha\cdot\mathbf{B}_\alpha)^2\right) \qquad \text{(3-D)}$$

Density, velocity, & kinematic viscosity calculated as

$$\rho = \sum_{\alpha=1}^{N} f_\alpha \qquad \rho v_\alpha = \sum_{\alpha=1}^{N} e_\alpha f_\alpha \qquad \nu = \frac{(\tau_f - .5)}{3}c^2\delta t$$

# Magnetic Field LBM Formulation

Magnetic induction eq. modeled using LB analogy because it's a conservative hyperbolic eq. [Dellar 2002]

$$\frac{\partial g_{\beta j}}{\partial t} + \Xi \cdot \nabla g_{\beta j} = -\frac{1}{\tau_g}\left(g_{\beta j} - g_{\beta j}^{(eq)}\right)$$

This recovers magnetic induction eq.

$$\frac{\partial \mathbf{B}}{\partial t} + \nabla \cdot \left(\mathbf{vB} - \mathbf{Bv}\right) + \nabla \cdot \left(\frac{\eta}{\mu_0} \nabla \mathbf{B}\right) = 0$$

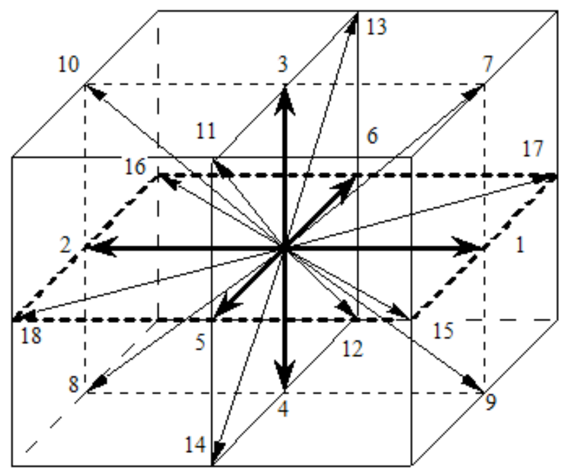
Dellar [2002] used equilibrium distribution function

$$\mathbf{g}_{\beta j}^{(eq)} = w_\beta \left[\mathbf{B}_j + \frac{4}{c}\left(v_i B_j - B_i v_j\right)\right]$$

Magnetic field & magnetic diffusivity calculated as

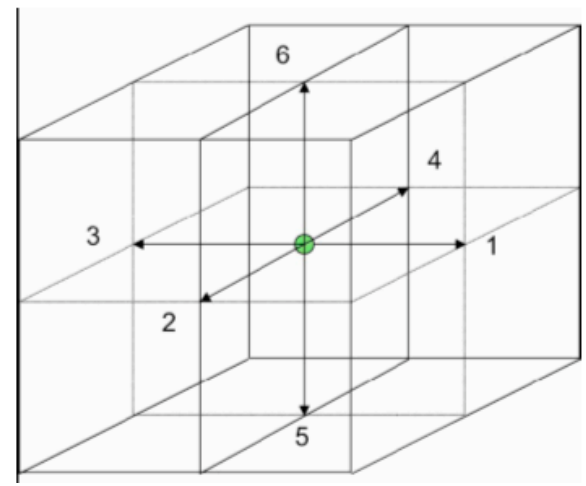
$$\mathbf{B}_j = \sum_{\beta=1}^{M} \mathbf{g}_{\beta j} \qquad \eta = \frac{\left(\tau_g - .5\right)}{4} c^2 \delta t \quad \text{(3-D)}$$

# Lattice Structures in MHD-LBM




Q19D3 Lattice Structure

$f_\alpha$




Q7D3 Lattice Structure

$\mathbf{g}_{\beta j}$

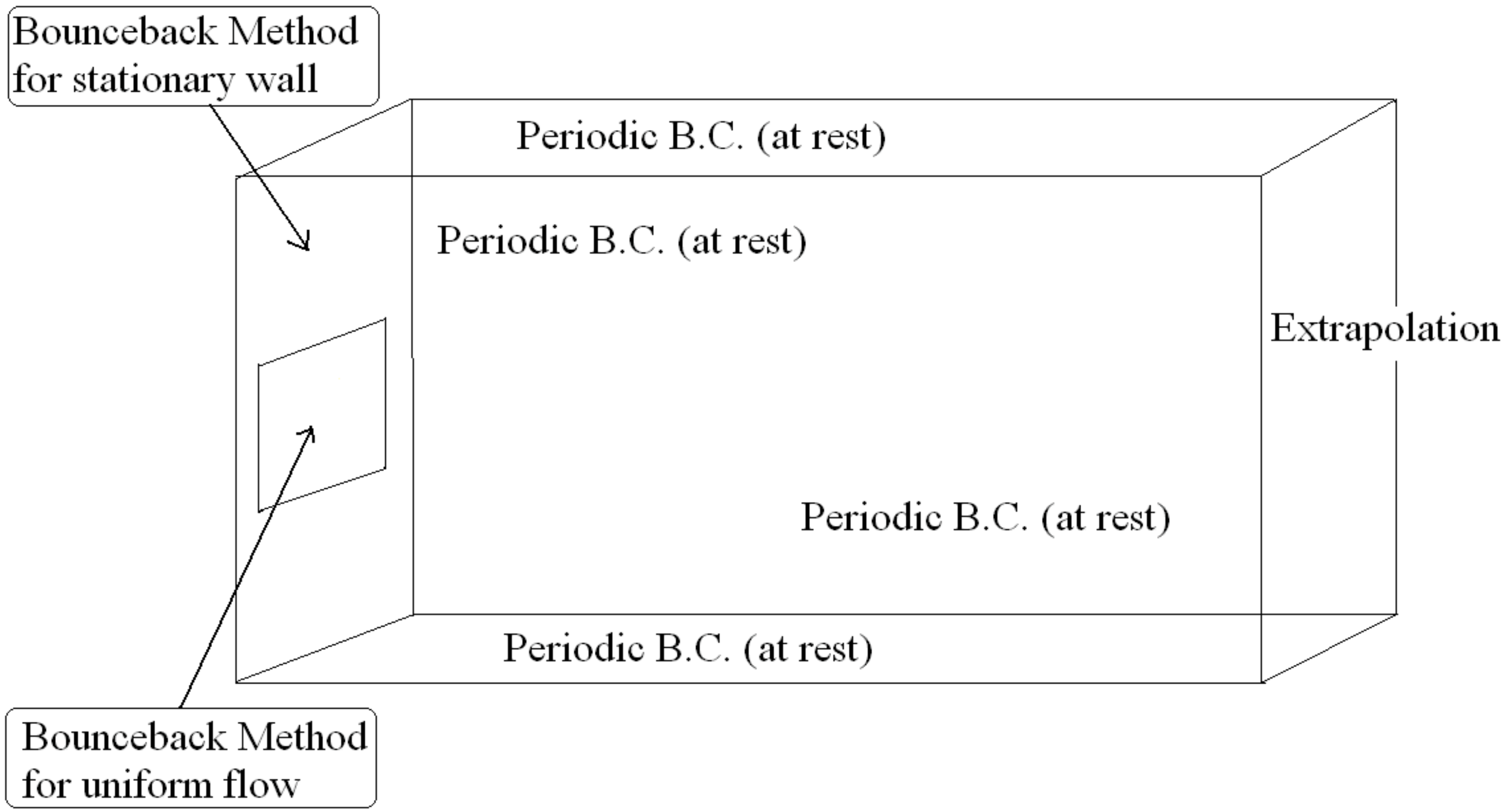
$$w_\alpha = \begin{cases} 1/3, & \alpha = 0 \\ 1/18, & \alpha = 1,2,\ldots 6 \\ 1/36, & \alpha = 7,8,\ldots 18. \end{cases}$$

$$W_\beta = \begin{cases} 1/4, & \beta = 0 \\ 1/2, & \beta = 1,2,\ldots 6 \end{cases}$$

# Velocity Field B.C.'s

Domain Dimension: 320x180x120

Outlet Dimension: 24x16

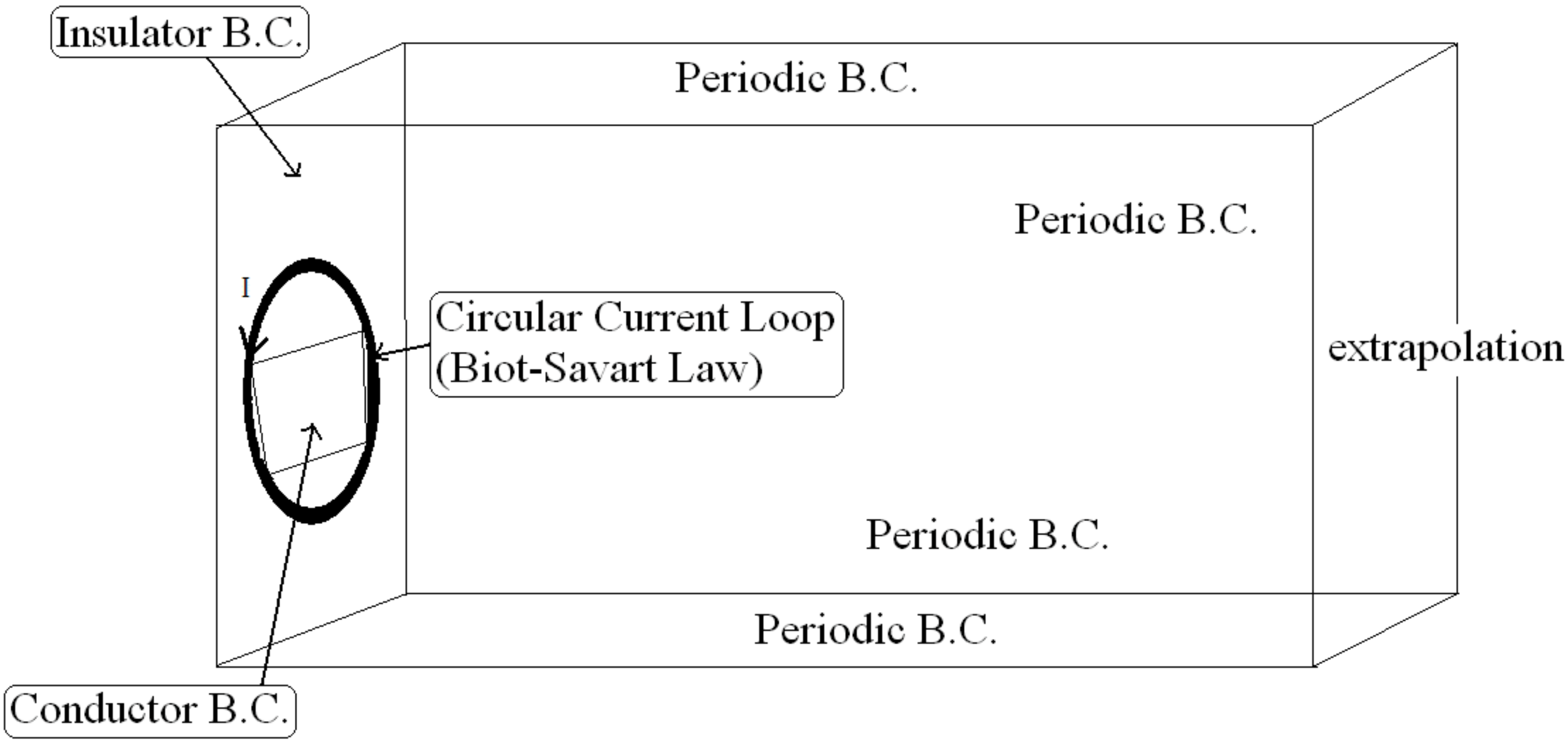



A constant uniform velocity field emits from the rectangular outlet of AR=1.5.

# Magnetic Field B.C.'s

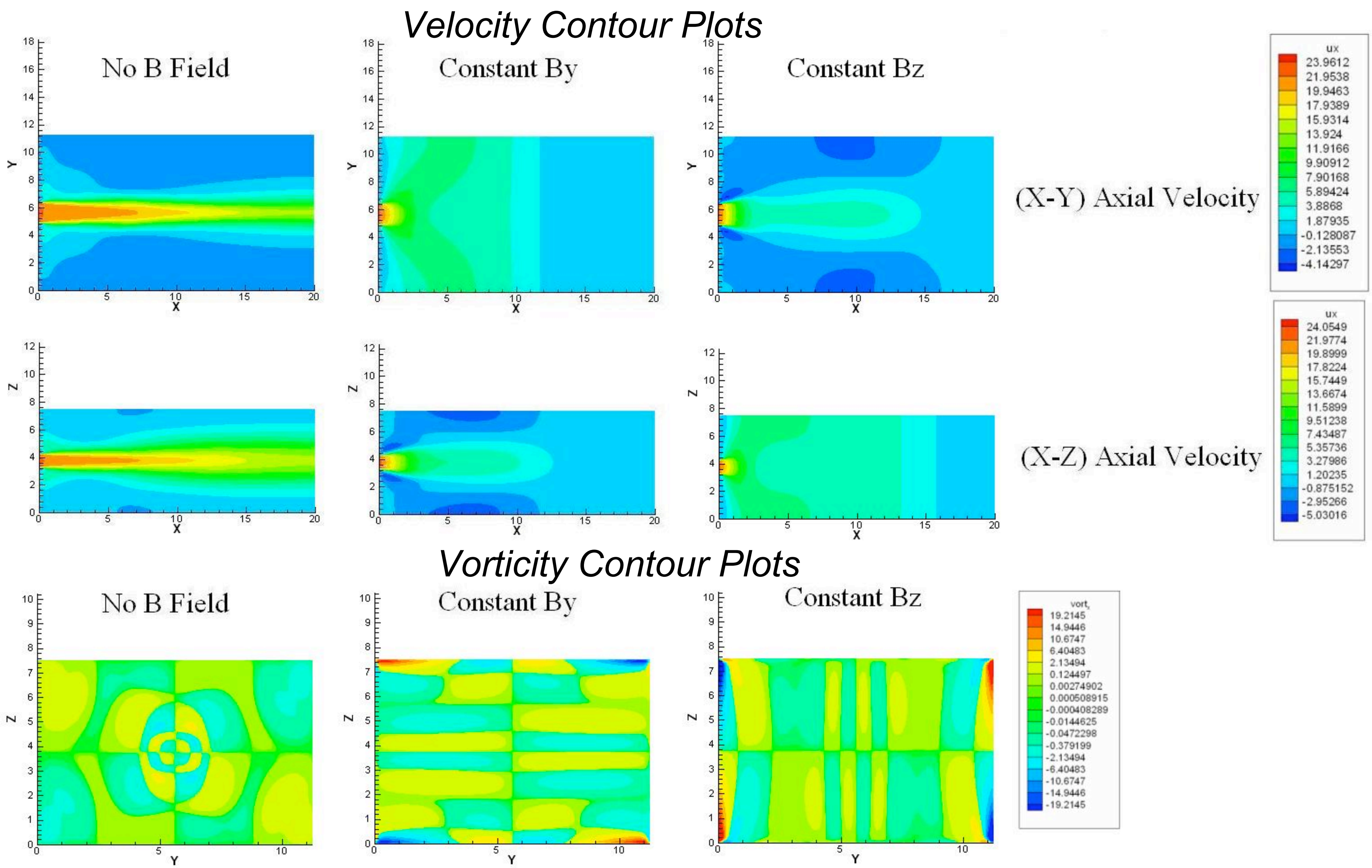
Domain Dimension: 320x180x120

Outlet Dimension: 24x16




Domain is under influence of a constant external magnetic field, created by a circular current loop around the rectangular outlet.

# Uniform Magnetic Field Influence on RJ




Trends correspond to observations in jet casting.

Correct physical trends provides credibility for MHD RJ simulations.

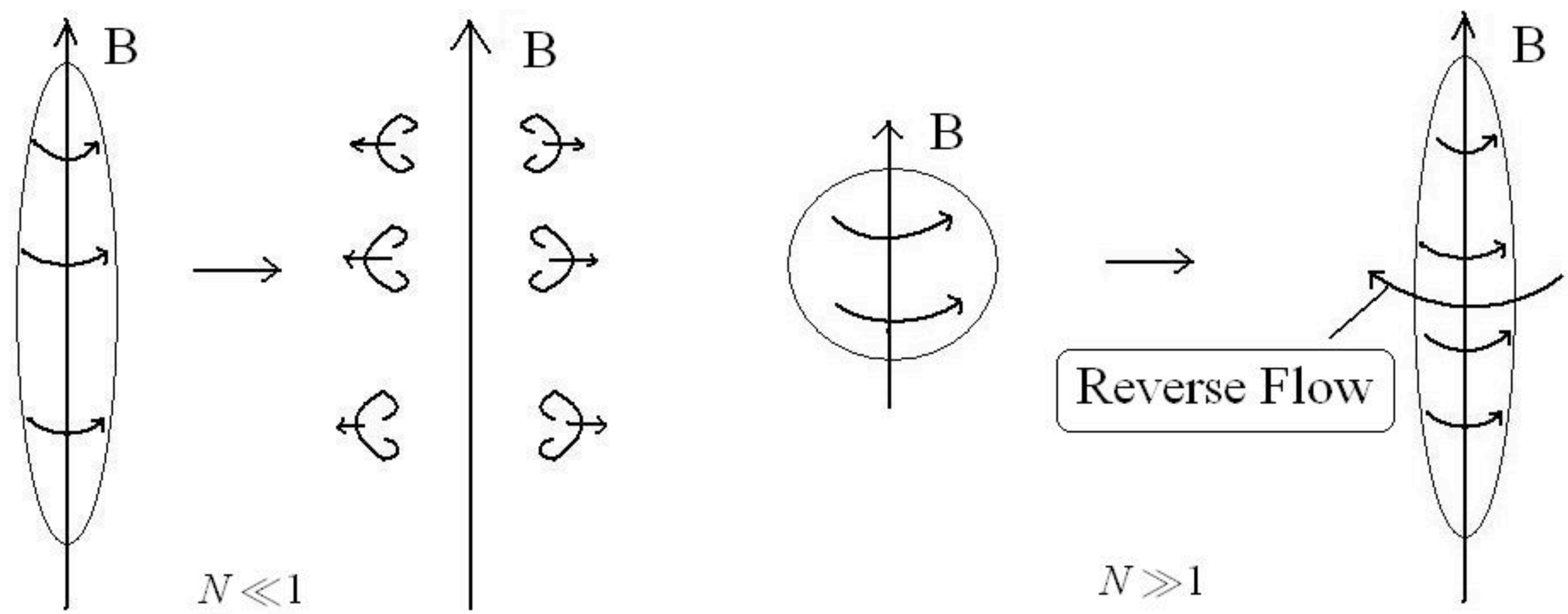
# Magnetic Field Effect on Vortices

$$\frac{\partial \boldsymbol{\omega}}{\partial t} + \mathbf{v} \cdot \nabla \boldsymbol{\omega} = \boldsymbol{\omega} \cdot \nabla \mathbf{v} + \boxed{\mathbf{B} \cdot \nabla \mathbf{j} - \mathbf{j} \cdot \nabla \mathbf{B}} + \nu \nabla^2 \boldsymbol{\omega}$$

**Vorticity Equation**

Vortex Stretching    Magnetic Tension Effect

**Attenuation of Parallel Vortices by uniformly applied B Field**



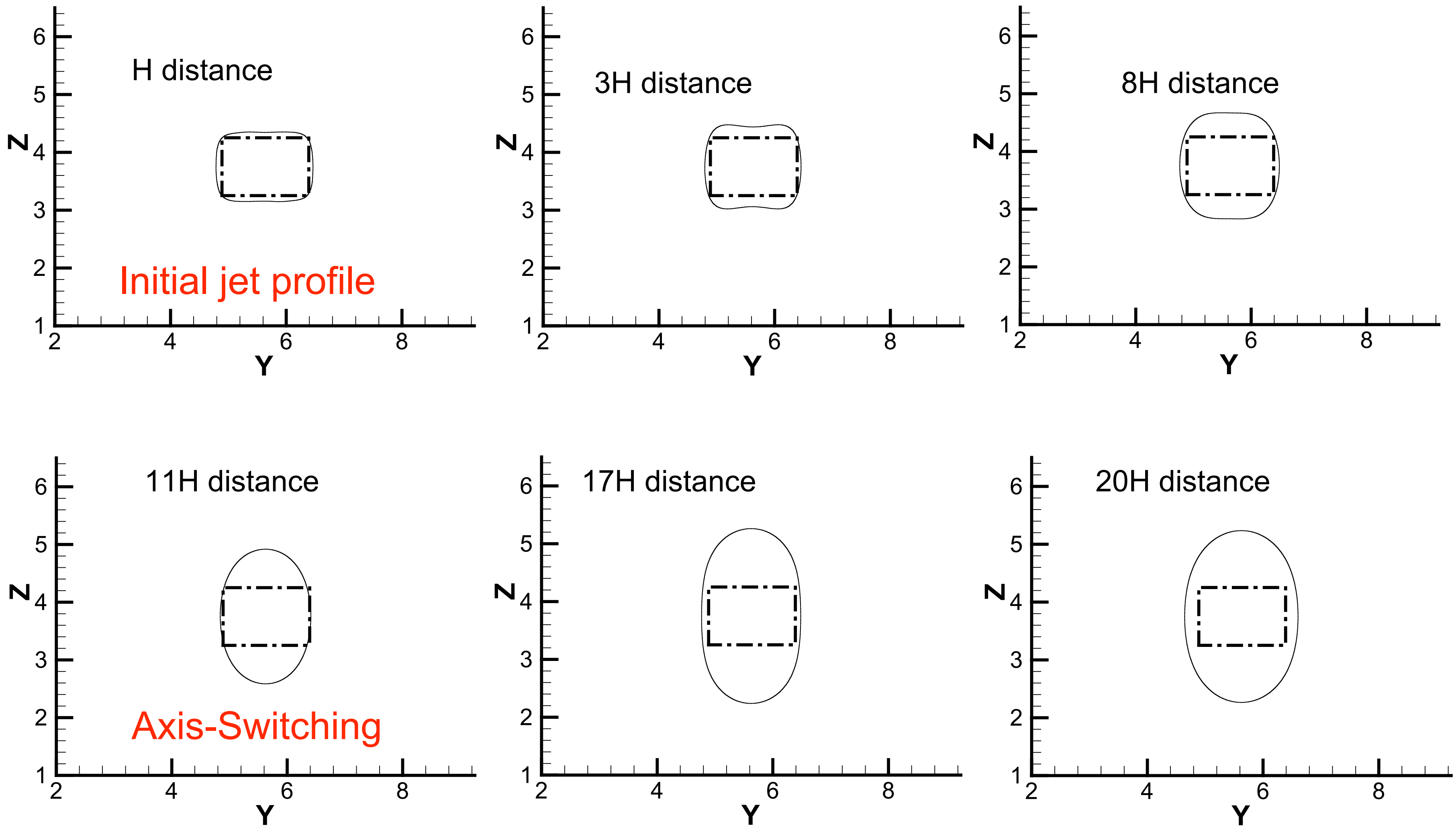

Small N damps vortices    Large N causes reverse flow

**These observations are for vortex generations, not Jet flow vortices.**

Davidson [2001]

# Axis-Switching (B=0)

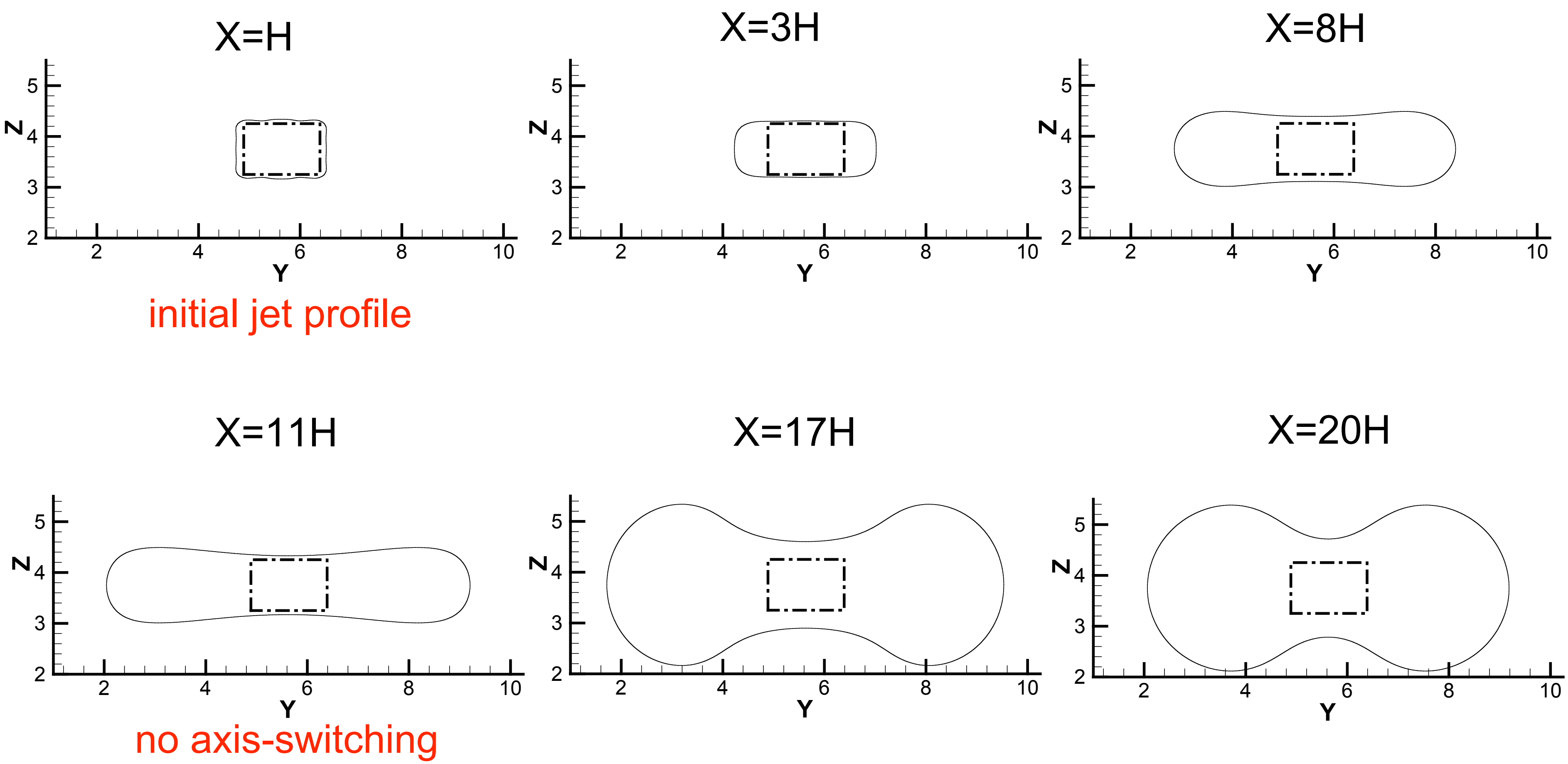
- Re=150, AR=1.5




Pictures are view of the streamwise plane (Y-Z) of the jet flow. Profiles @ v=v_max/2

# Attenuation of Axis-Switching (B ≠ 0, I=375 A)

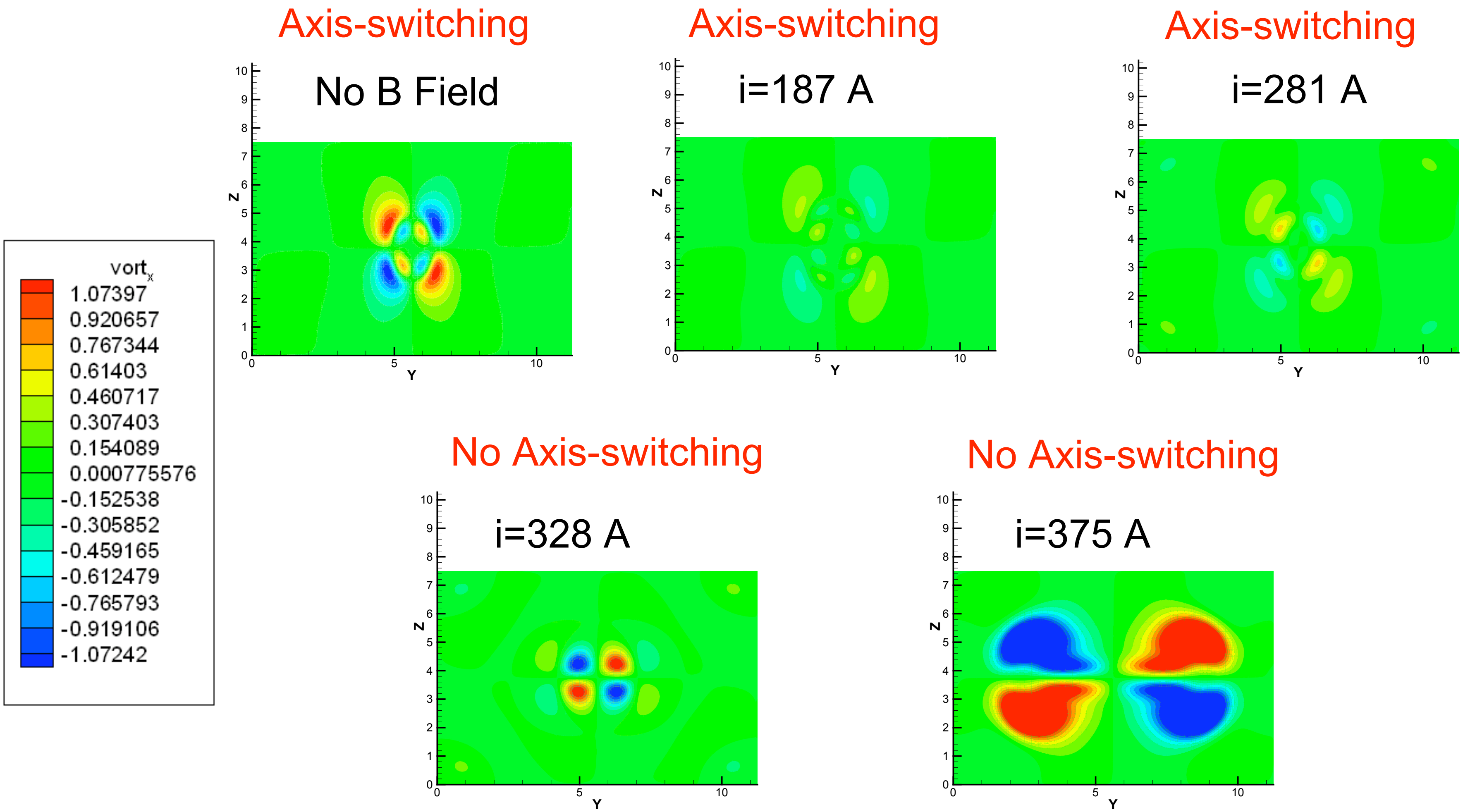
- Re=150. AR=1.5




Pictures are view of the streamwise plane (Y-Z) of the jet flow

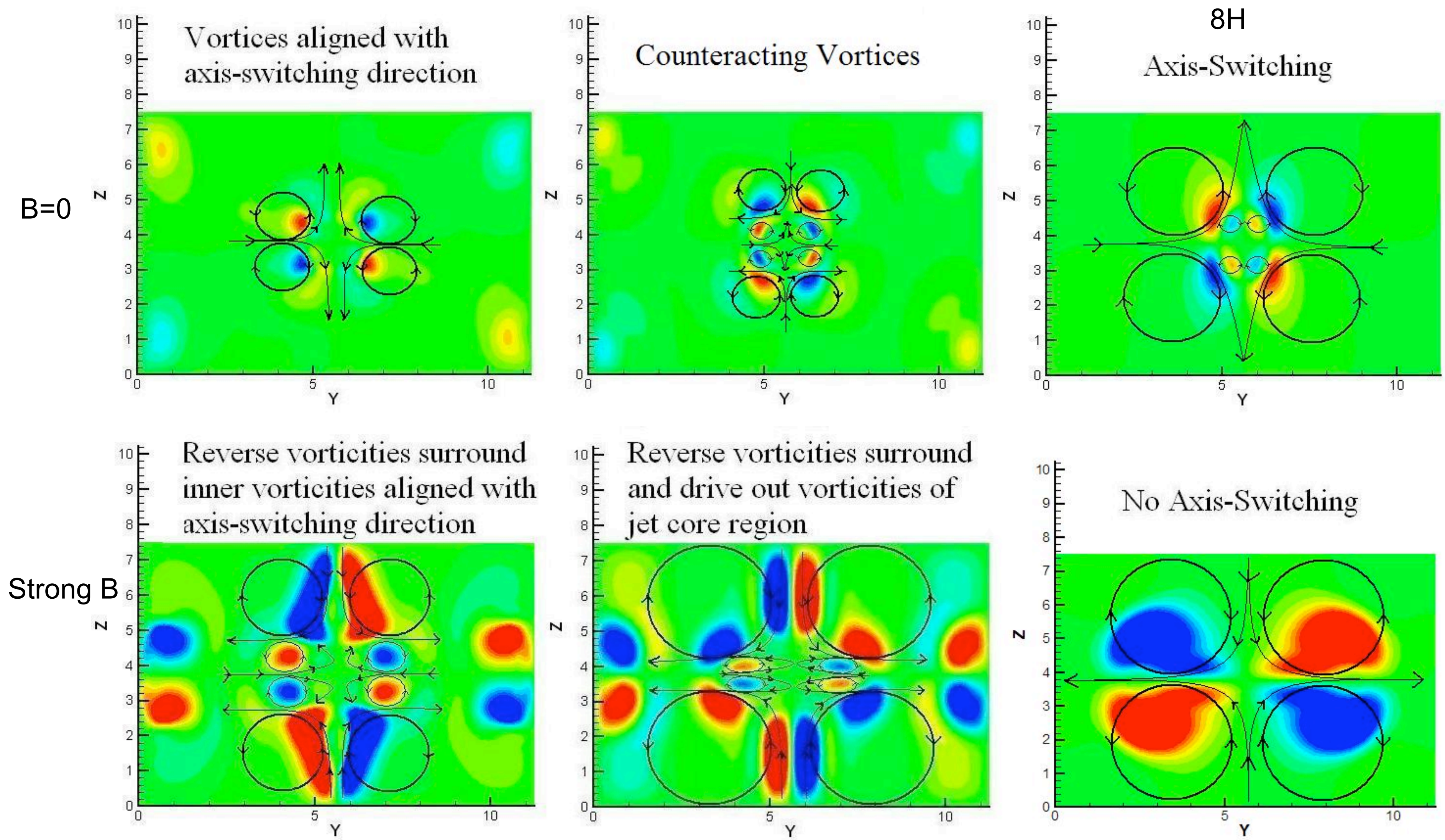
# Axial Vorticity Contour Plots at 11H

Pictures are view of the streamwise plane (Y-Z) of the jet flow

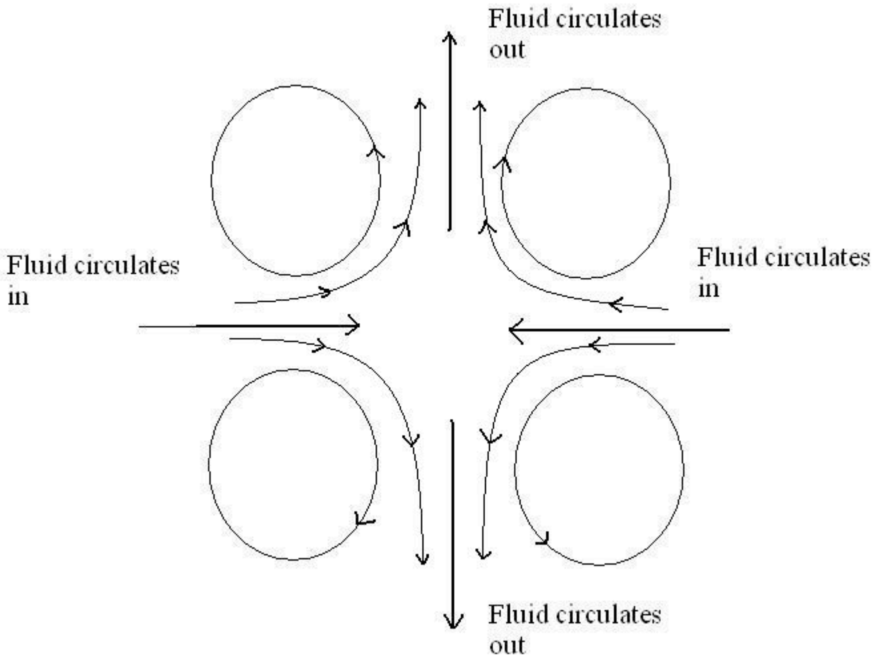



At first we see attenuation of vorticity, then vortices reverse alignment & strengthen, following trends presented by Davidson [2001].

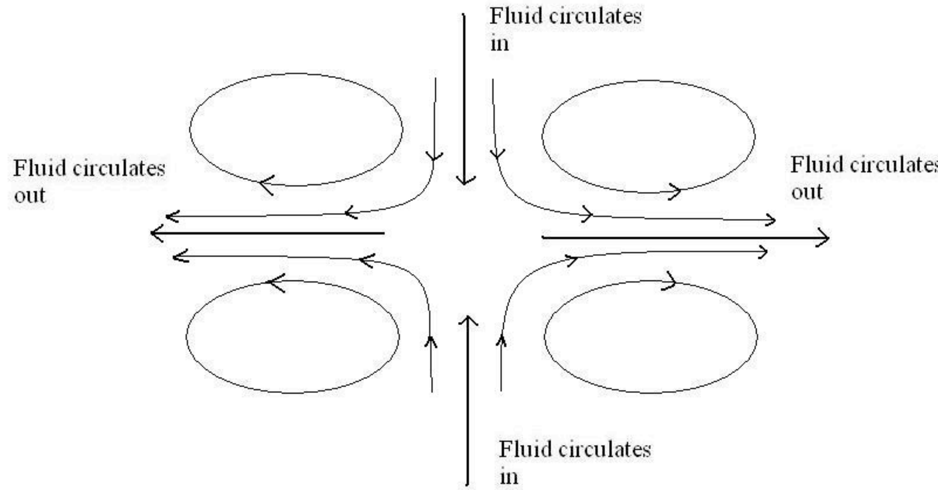
# Vortex Development in Axial Direction



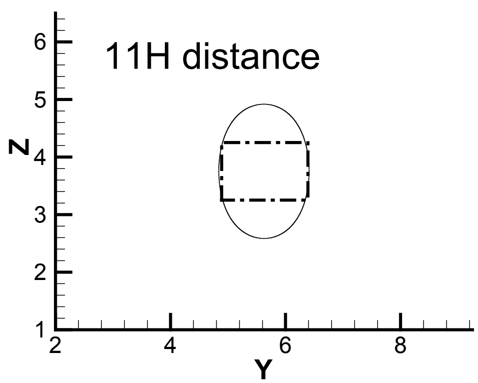
# Vorticity Circulation in Axis-Switching

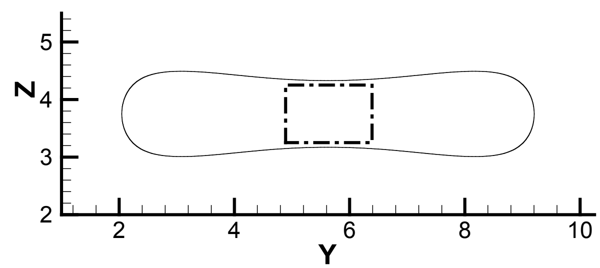



Fluid Circulation during Axis-Switching

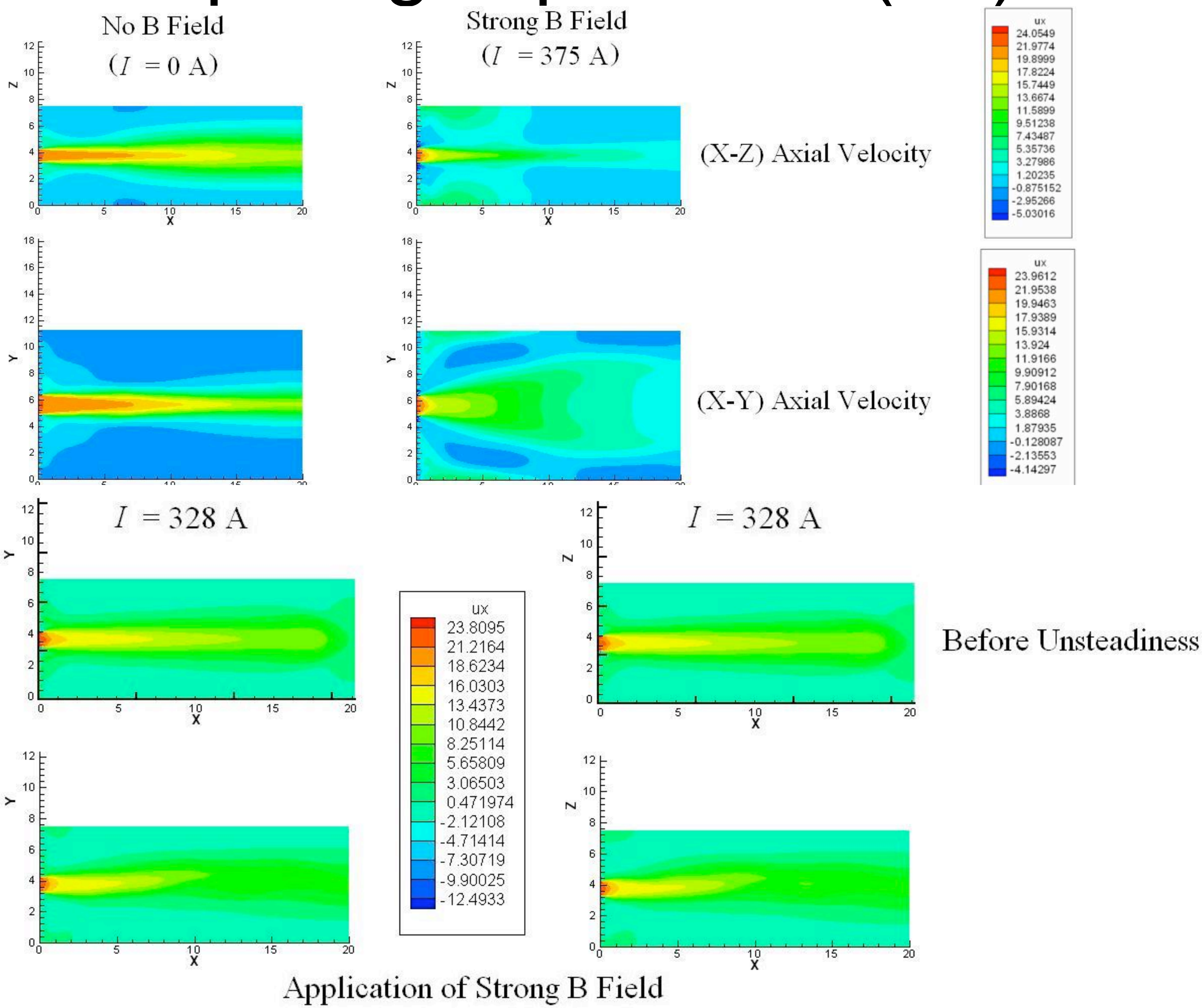



Fluid Circulation during prevention of Axis-Switching

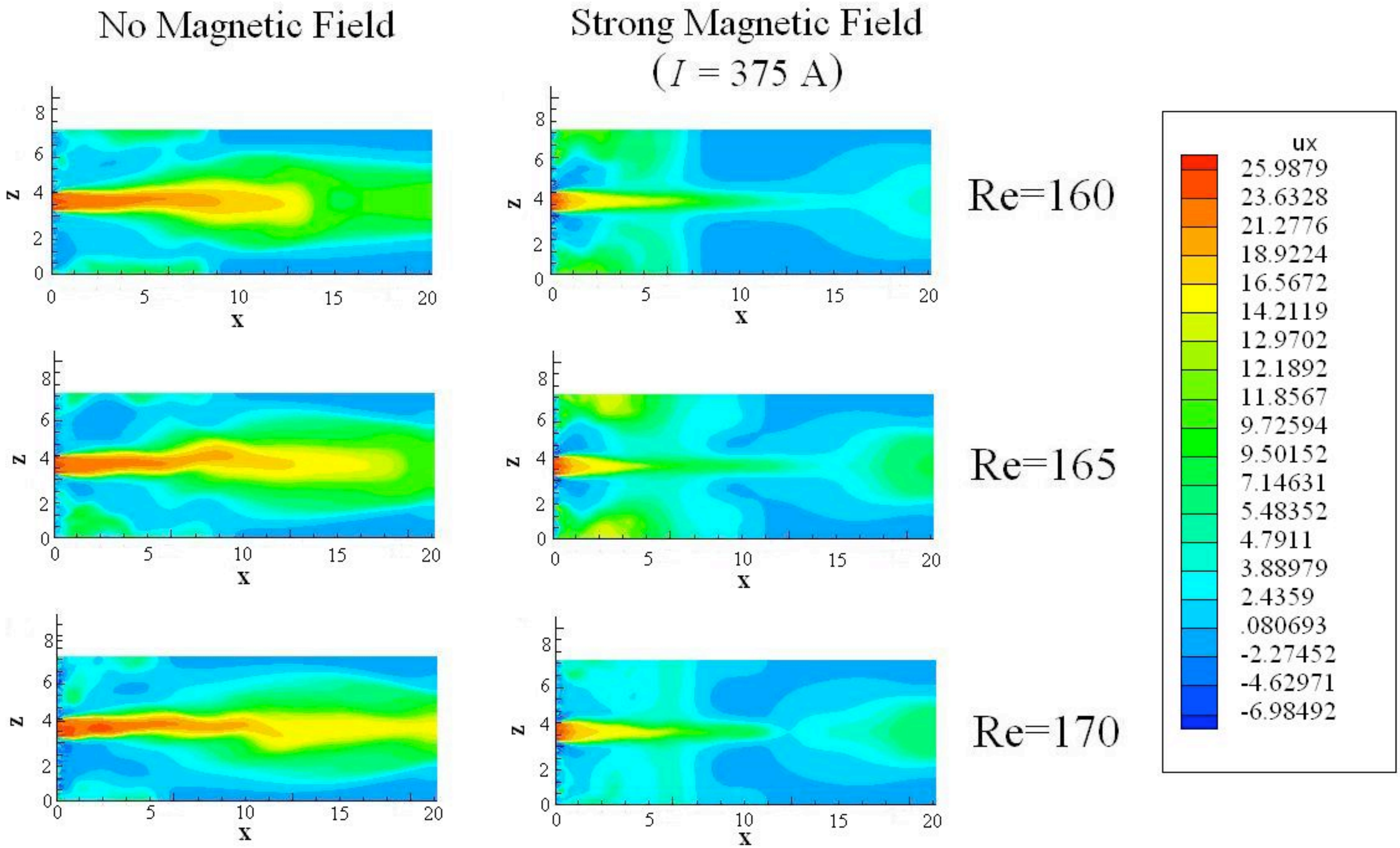


# Comparing Square Jet (SJ) & RJ



# Attenuation of Unsteadiness




Kelvin-Helmholtz instability mode attenuated due to strong axial magnetic field cpt.

Biskamp [2003] demonstrates that parallel magnetic field attenuates this instability.

# Conclusions

In RJ, major axis influences directionality of plume structure.

Jet profile elongates in major axis direction when strong axis-symmetric magnetic field is applied.

External magnetic field influence on a jet flow causes:

- Attenuation of the velocity field
- Vortex Attenuation and Reversal
- Kelvin-Helmholtz instability is attenuated

These effects lead to prevention of axis-switching and unsteadiness in Rectangular Jets.

# References


- Nanotube
  - Sigmund, Bell & Bergstrom [2000]
- 2D, transverse fields, flow control
  - Hunt and Leibovich [1967]; Bobashev, Golovachov & van Wie [2003]; Nishihara, Bruzzese, Adamovich, Udagawa & Gaitonde [2007]; Moreau [1963]; Moffatt & Toomre [1967]; Macheret, Shneider, & Miles [2002]
- Axis-switching of laminar RJ
  - Yu & Girimaji [2005]